

Comment on “Taming the Pion Cloud of the Nucleon”

Chueng-Ryong Ji^a, W. Melnitchouk^b, A. W. Thomas^c

^a Department of Physics, North Carolina State University, Raleigh, North Carolina 27692, USA

^b Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA

^c CSSM and CoEPP, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005, Australia

In a recent Letter, Alberg and Miller (AM) [1] presented a calculation of the pion contributions to the self-energy of the nucleon, in an attempt to better constrain the role of the pion cloud in the $\bar{d} - \bar{u}$ asymmetry in the proton sea. The self-energy Σ was computed from a pseudoscalar (PS) pion–nucleon interaction $(\bar{\psi}_N \gamma_5 \vec{\tau} \psi_N \cdot \vec{\phi}_\pi)$, with the claim that the result would be equivalent to that with the more usual pseudovector (PV) form $(\bar{\psi}_N \gamma_\mu \gamma_5 \vec{\tau} \psi_N \cdot \partial^\mu \vec{\phi}_\pi)$. The PV theory is consistent with the chiral symmetry properties of the strong interactions, as embodied for example in chiral perturbation theory, whereas the PS coupling requires in addition a scalar field to restore chiral invariance [2].

In this Comment we demonstrate that the PV and PS pion–nucleon couplings do in fact lead to different results for the self-energy. In particular, for the model-independent, leading nonanalytic (LNA) behavior of the self-energy, the PV theory yields the well-known m_π^3 dependence in the chiral limit, while the PS interaction involves an additional, lower-order term $\sim m_\pi^2 \log m_\pi^2$ [3]. To identify the origin of the difference, we can express the total self-energy for the PS coupling Σ^{PS} in terms of the PV self-energy Σ^{PV} and a contribution from the end-point region corresponding to $k^+ = 0$, $\Sigma^{\text{PS}} = \Sigma^{\text{PV}} + \Sigma_{\text{end-pt}}^{\text{PS}}$. Using the Goldberger-Treiman relation to relate the πNN coupling to the axial charge of the nucleon g_A and the pion decay constant f_π , the end-point contribution can be written as

$$\Sigma_{\text{end-pt}}^{\text{PS}} = \frac{3ig_A^2 M}{2f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{F^2(k^2, (p-k)^2)}{k^2 - m_\pi^2 + i\epsilon}, \quad (1)$$

where, in analogy with AM, we introduce a πNN form factor F which suppresses contributions from short distances. Although our results do not depend on the details of the short-distance πN interaction, for generality we keep the dependence of the form factor on the invariant mass of both the intermediate state pion and nucleon.

When performing the k^- integration, it is crucial to realize that the pion pole depends not only on the sign of k^+ , but also effectively runs to infinity as $k^+ \rightarrow 0$. Keeping this runaway pole in the k^- contour integration is the key to the difference between Σ^{PS} and Σ^{PV} . In Ref. [1], AM consider only $k^+ > 0$ and $k^+ < 0$, and omit the contribution from the end-point $k^+ = 0$. Including the pole at infinity, one finds [3]

$$\Sigma_{\text{end-pt}}^{\text{PS}} = \frac{3g_A^2 M}{16\pi^2 f_\pi^2} \int_0^\infty dt \frac{\sqrt{t} F^2(m_\pi^2, -t)}{\sqrt{t + m_\pi^2}}, \quad (2)$$

where the form factor is evaluated at the pion pole.

To evaluate the contribution in Eq. (2) explicitly, we can use a dipole parametrization for the dependence of the form factor on the nucleon virtuality, $F(m_\pi^2, -t) = ((\Lambda^2 - M^2)/(\Lambda^2 + t))^2$, with Λ a mass parameter. Following AM, we define $a = m_\pi^2/M^2$, $b = \Lambda^2/M^2$ and find

$$\Sigma_{\text{end-pt}}^{\text{PS}} = \frac{3g_A^2 M}{64\pi^2 f_\pi^2} \left\{ \sqrt{b(a-b)}(a-4b)(3a+2b) + 3a(a^2 - 4ab + 8b^2) \tan^{-1} \sqrt{\frac{a}{b} - 1} \right\} \frac{(b-1)^4}{6(a-b)^{\frac{7}{2}} b^{\frac{5}{2}}}. \quad (3)$$

Expanding the term proportional to $\tan^{-1} \sqrt{a/b - 1}$ about $a = 0$, the LNA term is found to be $\sim a \log a$, which is of lower order than the LNA term for the PV coupling $\sim m_\pi^3$. The lowest nonanalytic terms for the total PS self-energy are then given by [3]

$$\Sigma_{\text{nonanal.}}^{\text{PS}} = \frac{3g_A^2}{32\pi f_\pi^2} \left(\frac{M}{\pi} m_\pi^2 \log m_\pi^2 - m_\pi^3 - \frac{m_\pi^4}{2\pi M^2} \log \frac{m_\pi^2}{M^2} + \mathcal{O}(m_\pi^5) \right). \quad (4)$$

Note that this result is independent of the short-distance part of the πNN interaction (or the form factor F), and can be verified using either light-front, time-ordered or covariant perturbation theory [3].

By using the PS theory and omitting the end-point singularities at $k^+ = 0$, AM happen to obtain the same result as given by the PV theory. However, this *ansatz* will not give the correct PV result for other quantities, such as the pion momentum distribution, f_π^N [1, 4]. For example, the moment of f_π^N (which corresponds to the pion loop contribution to the vertex renormalization Z_1^π) in the PS theory gives for the LNA term a value $4/3$ larger than for the PV theory [5, 6], with the difference given by an end-point contribution in the PV case. The pseudoscalar coupling therefore cannot in general be used if one wishes to ensure consistency with the chiral properties of QCD, which are respected by the pseudovector πN coupling.

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